# Doubly-efficient zkSNARKs without trusted setup


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# zkSNARK

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- Non-interactive ...and it can be written down...
- (Publicly verifiable) ...so that anyone can check it.

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# Our contributions

➔ We design and implement *Hyrax*, a zkSNARK for "parallel" arithmetic circuit satisfiability: for $\mathcal{V}$'s input $x$, $\exists w : \mathcal{C}(x, w) = 1$ (and $\mathcal{P}$ knows $w$)

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> Hyrax is one useful point in a large tradeoff space

# Roadmap

1. General-purpose ZK proof systems
2. Hyrax at a high level
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# General-purpose ZK proof systems for NP

On input $x$, $\mathcal{P}$ convinces $\mathcal{V}$ that $\Phi(x, w) = 1$
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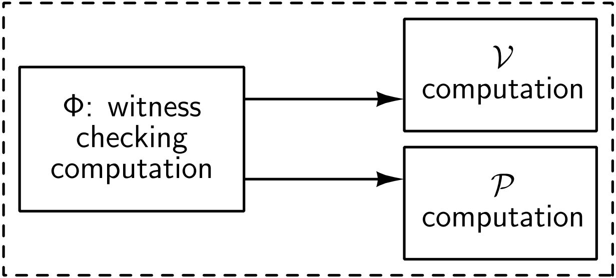

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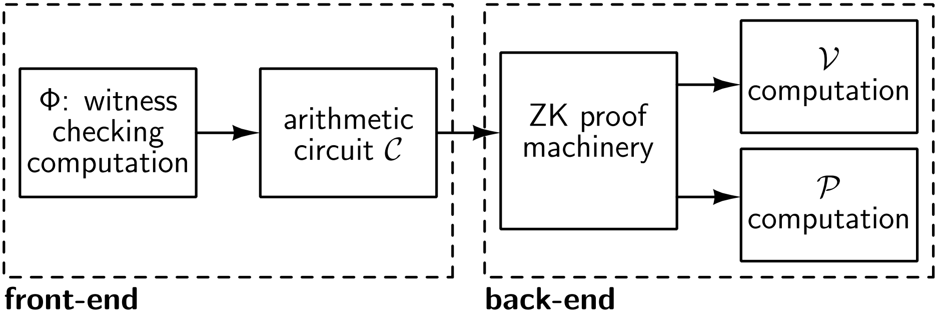

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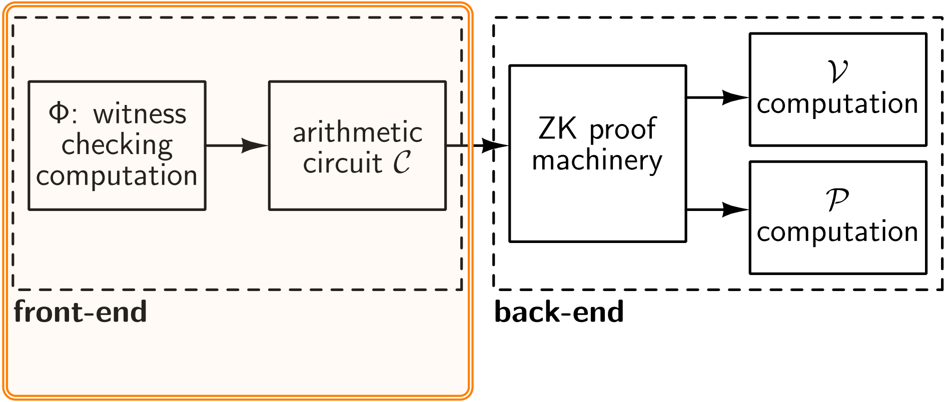

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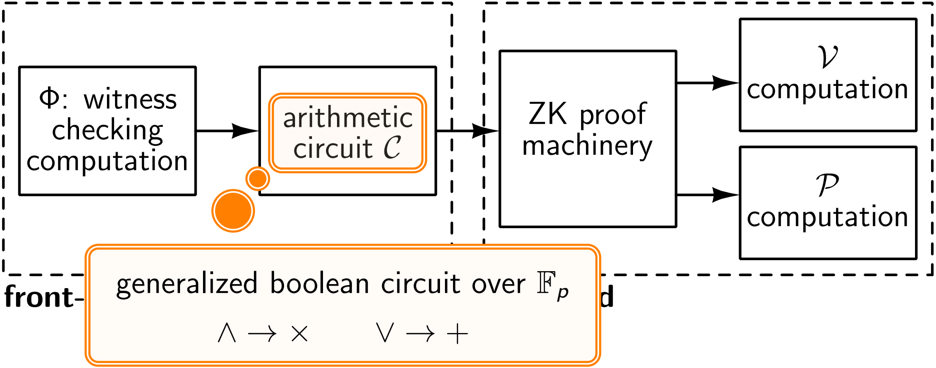

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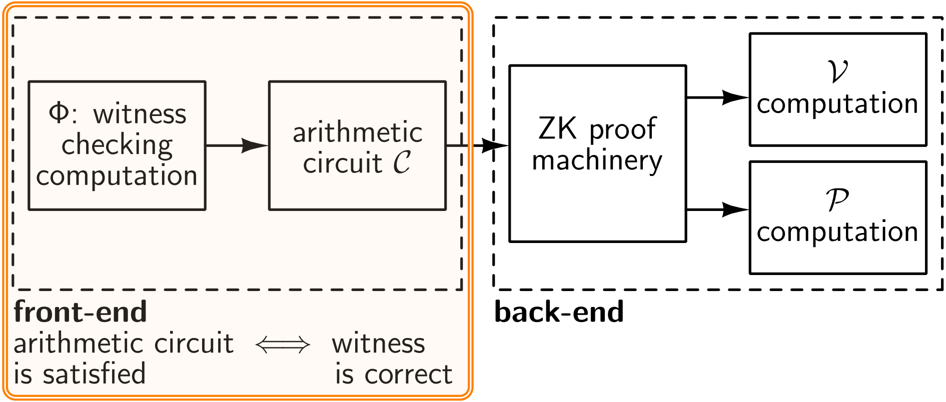

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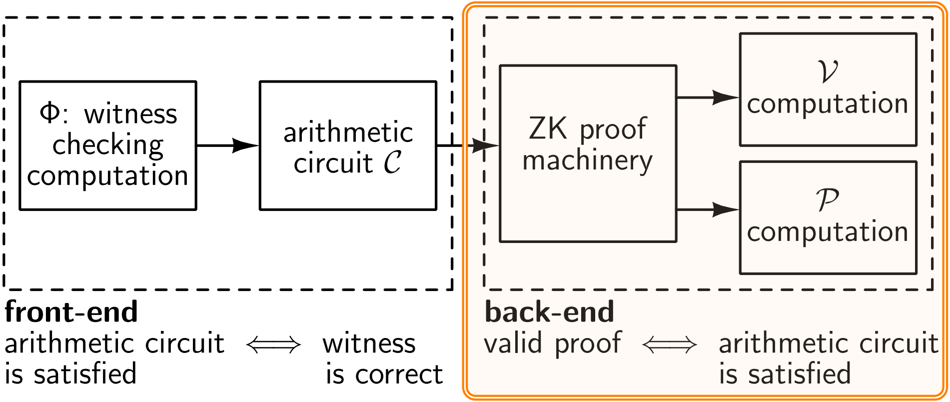

# Existing systems use a wide range of proof machinery

## Linear PCPs [IKO07,Gro09,Gro10,BG12,Lip12,BCIOP13,GGPR13,. . . ]

- Pinocchio [PGHR13], libsnark [BCTV14]

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## Short PCPs [Kil94,Mic00,BS08,BCN16,RRR16,BBC+17,BBHR17,...]

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Hyrax builds on the interactive proofs of GKR/CMT

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High-level idea: Replace each of $\mathcal{P}$'s messages in the IP with a *commitment* to the message; $\mathcal{V}$ runs checks "under the commitments."

# Cryptographic commitments

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We also require a *linear homomorphism*, $\odot$:
given $C_0 \leftarrow \mathrm{Com}(m_0)$, $C_1 \leftarrow \mathrm{Com}(m_1)$, we have

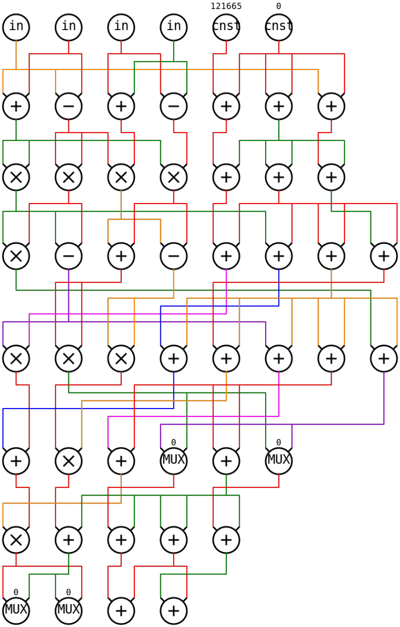
$$C_0 \odot C_1 \triangleq \mathrm{Com}(m_0 + m_1)$$

$$C_1^k \triangleq C_1 \odot \cdots \odot C_1 = \mathrm{Com}(k \cdot m_1)$$

The Pedersen commitment has this property.

# GKR08: IP for arithmetic circuit evaluation (non-ZK)

Witness checker must be expressed as a *layered* AC.



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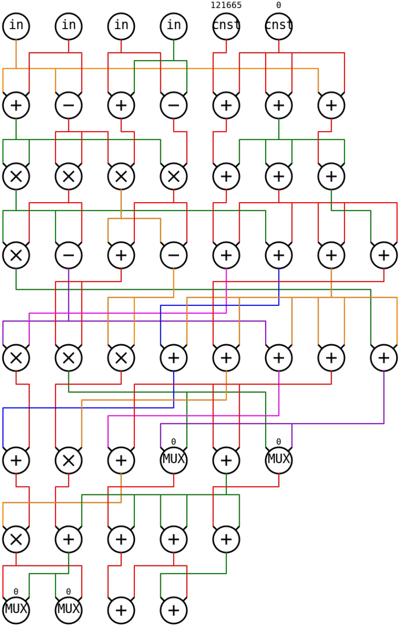


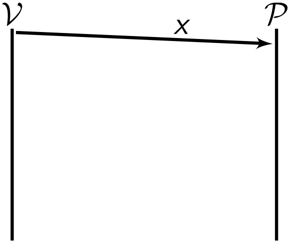

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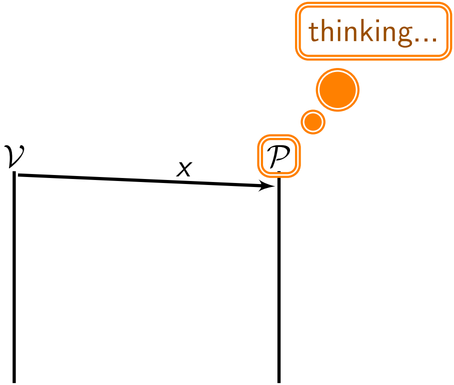


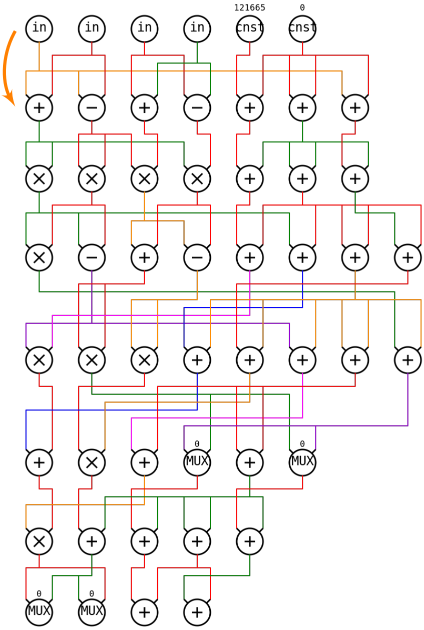

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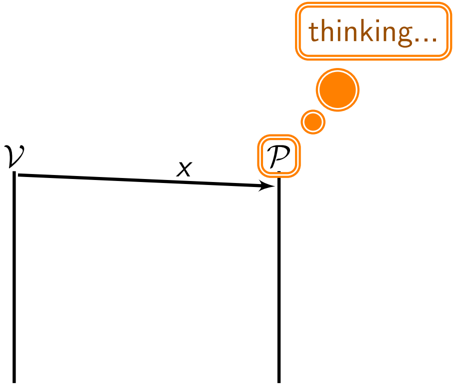


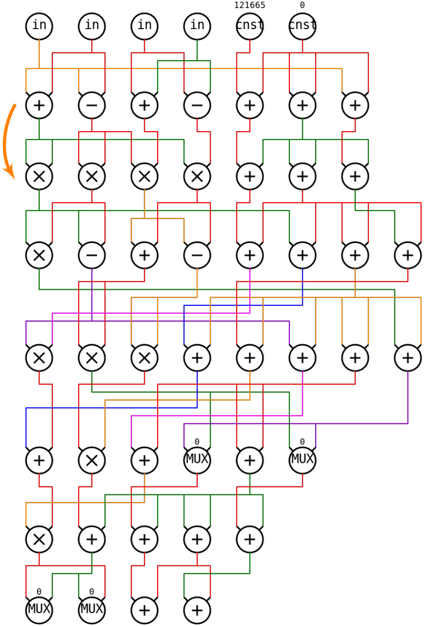

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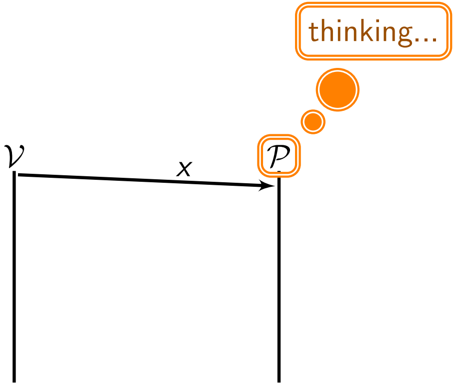


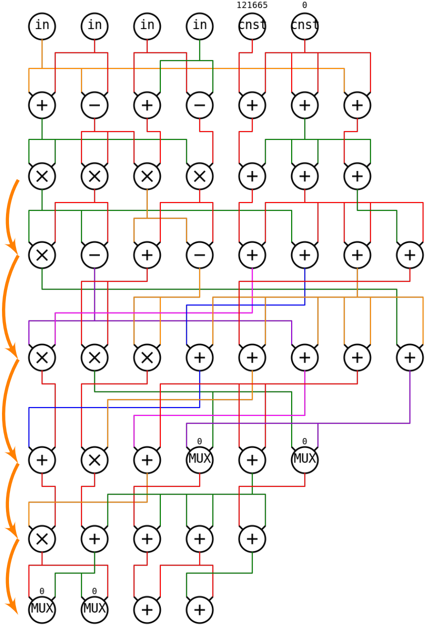

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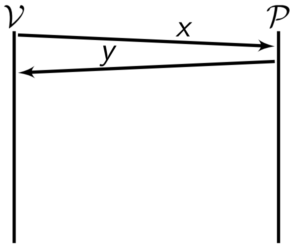


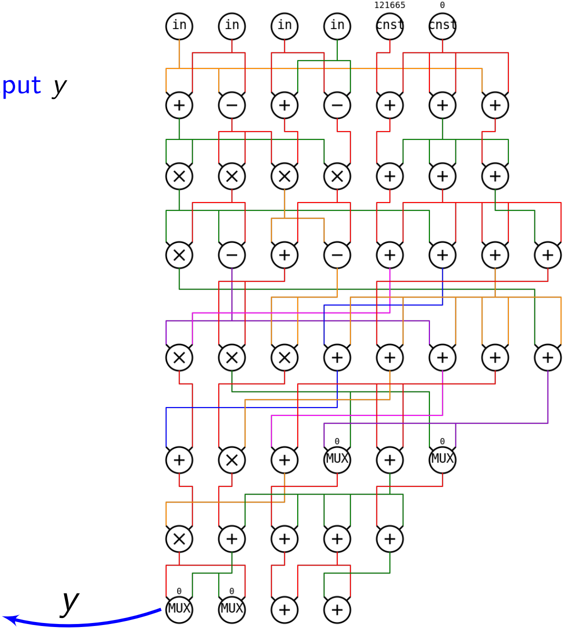

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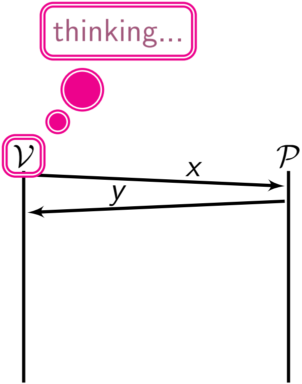


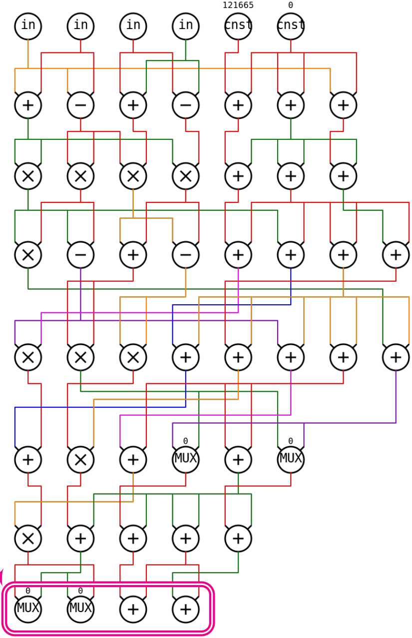

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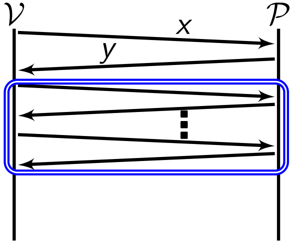


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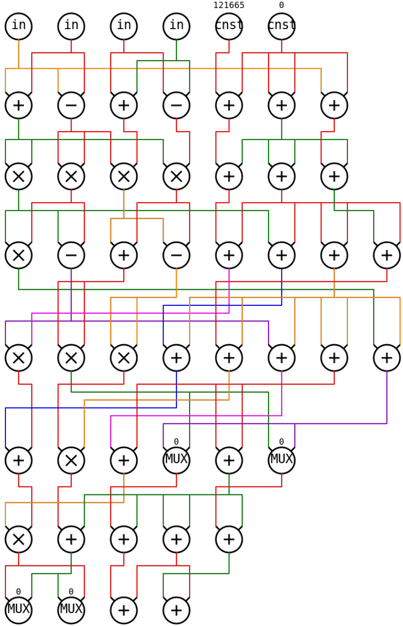

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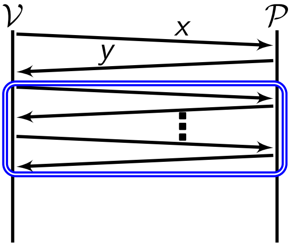


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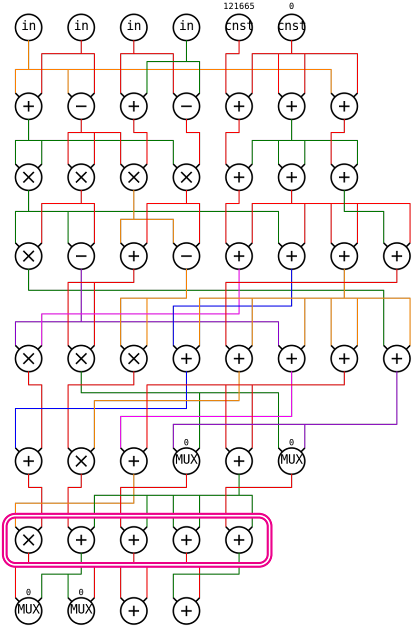

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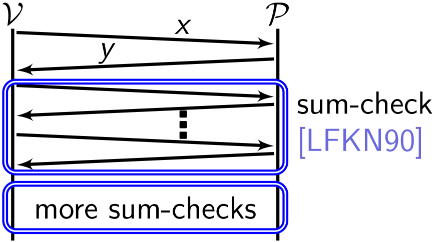


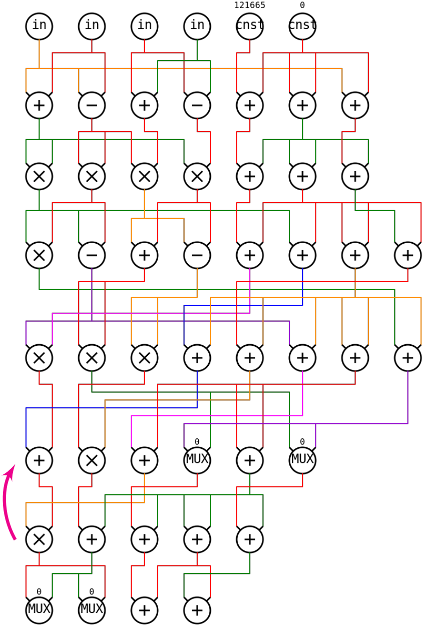

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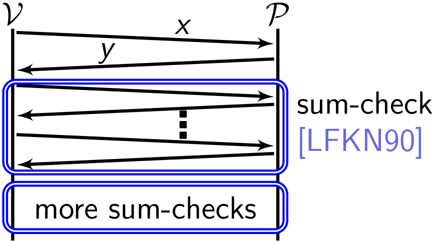


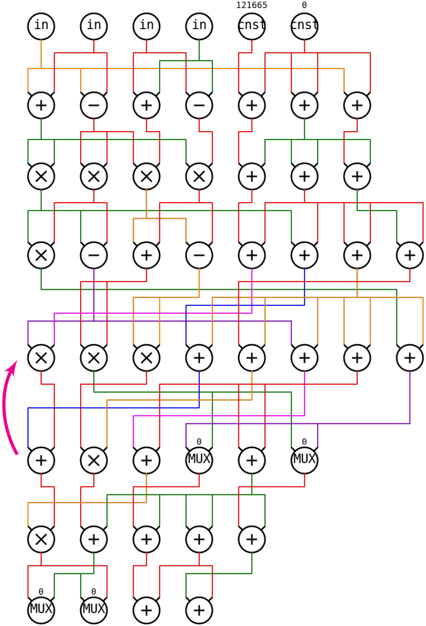

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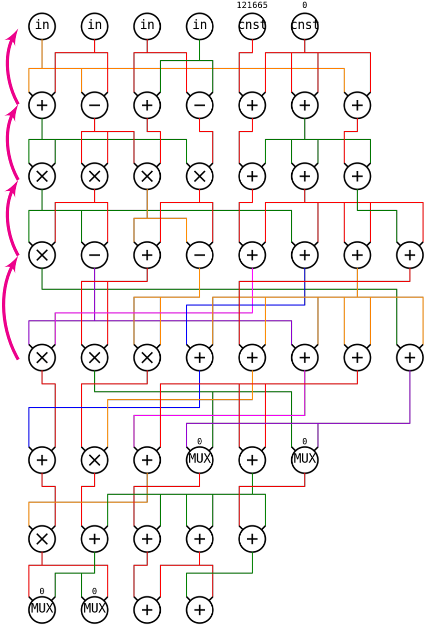


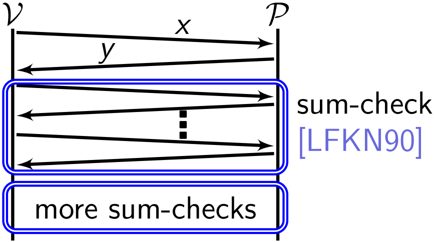

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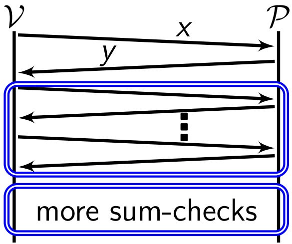
5. $\mathcal{V}$ iterates, gets claim about inputs, which it can check

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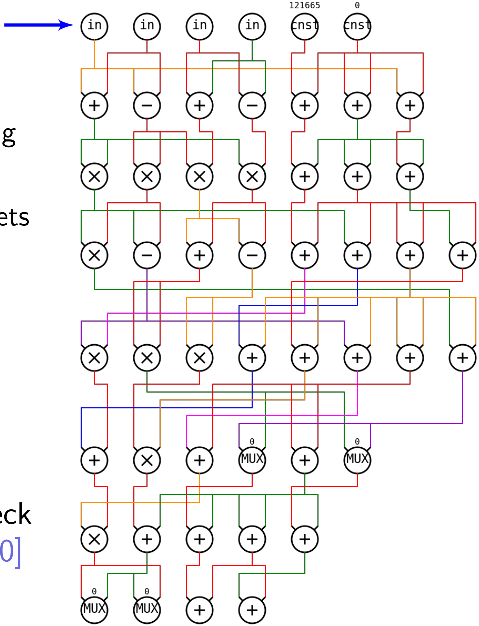

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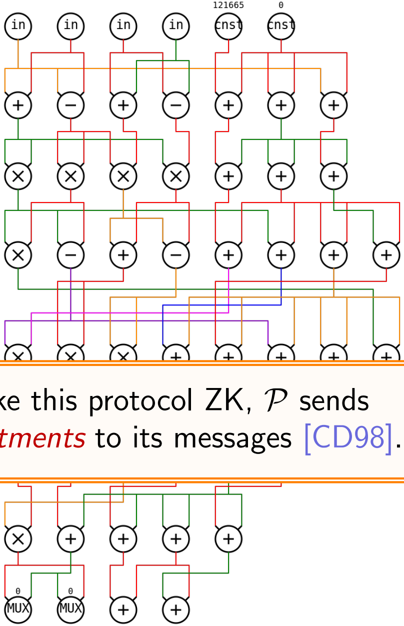
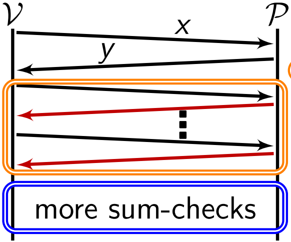
4. $\mathcal{V}$ engages $\mathcal{P}$ in a sum-check, gets claim about second-last layer
5. $\mathcal{V}$ iterates, gets claim about inputs, which it can check




To make this protocol ZK, $\mathcal{P}$ sends *commitments* to its messages [CD98].




sum-check [LFKN90]

# GKR08: IP for arithmetic circuit evaluation (with ZK)

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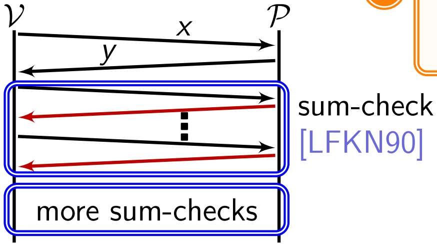


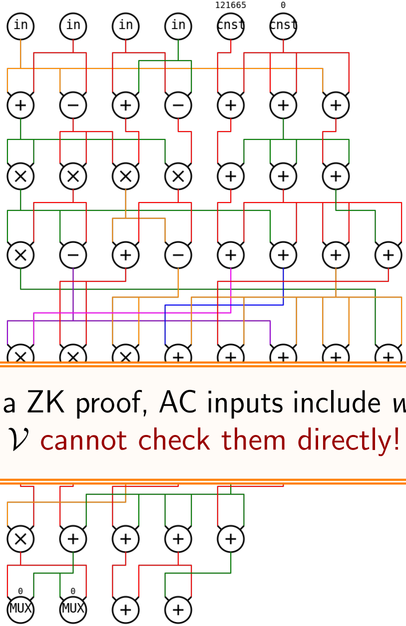


In a ZK proof, AC inputs include $w$, so $\mathcal{V}$ cannot check them directly!

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$\mathcal{V}$'s final check is to evaluate a polynomial $\widetilde{m}$ that encodes input $x$ and witness $w$.

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Hyrax uses a new polynomial commitment scheme tailored to *multilinear*$^\star$ polynomials like $\widetilde{m}$

$^\star$multivariate, linear in each variable

# A polynomial commitment for $\widetilde{m}$

$$\widetilde{m}(r) \triangleq L \cdot T \cdot R^T$$

$\mathcal{V}$ can compute $L$ and $R$ from $r$, and

$$T \triangleq \begin{bmatrix} w_0 & w_\ell & \cdots & w_{\ell^2-\ell} \\ w_1 & w_{\ell+1} & \cdots & w_{\ell^2-\ell+1} \\ \vdots & \vdots & \ddots & \vdots \\ w_{\ell-1} & w_{2\cdot\ell-1} & \cdots & w_{\ell^2-1} \end{bmatrix}$$

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✗ Proof size and $\mathcal{V}$ time are both $\mathrm{O}(|w|)$!

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$$T_0 = \text{Com}(w_0, w_\ell, \ldots, w_{\ell^2-\ell}) \quad [Gro09]$$

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Pedersen commitments: vector-wise homomorphism.

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1. $\mathcal{V}$ uses homomorphism to compute $\mathrm{Com}(L \cdot T)$.
2. $\mathcal{P}$ sends a commitment to an evaluation of $\widetilde{m}(r)$
3. $\mathcal{P}$ uses a *dot-product argument* to convince $\mathcal{V}$ that $\mathrm{Com}(\widetilde{m}(r))$ is consistent with $R$ and $\mathrm{Com}(L \cdot T)$.

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Gir$^{++}$ IP: Giraffe [WJBsTWW17] plus a tweak [CFS17]
➔ reduces proof size

# Roadmap

1. General-purpose ZK proof systems
2. Hyrax at a high level
3. Evaluation

# Evaluation overview

Baselines:

- BCCGP-sqrt [BCCGP16]—re-implemented
- Bulletproofs [BBBPWM18]—re-implemented
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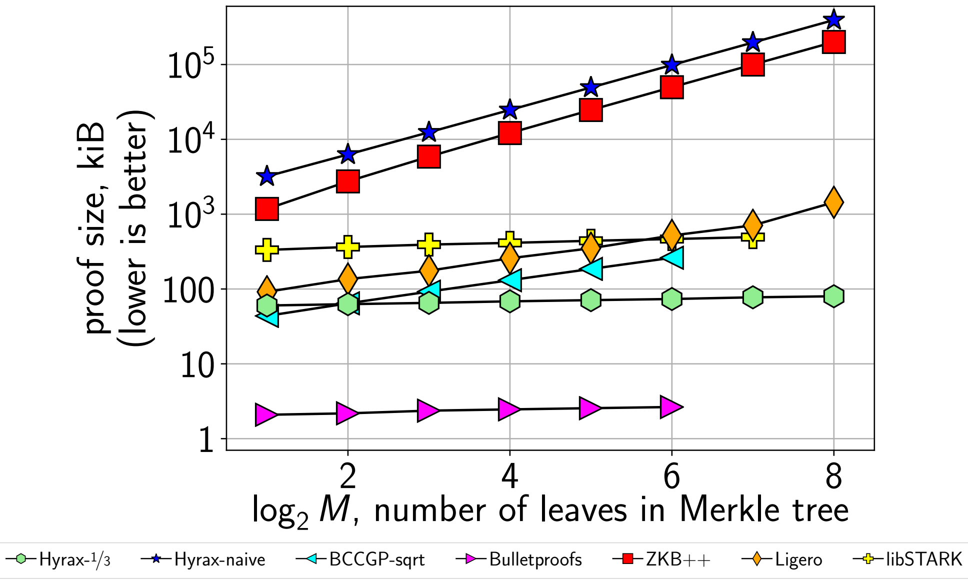
Benchmark: SHA-256 Merkle tree,
varying number of leaves

# Proof size



proof size, kiB
(lower is better)
1
10
100
$10^3$
$10^4$
$10^5$
2
4
6
8
$\log_2 M$, number of leaves in Merkle tree
Hyrax-1/3
Hyrax-naive
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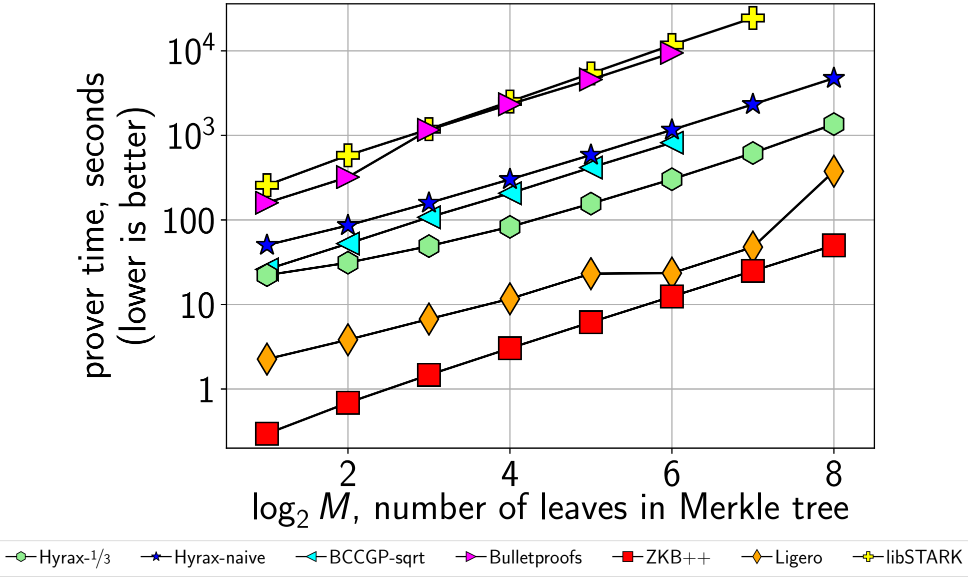
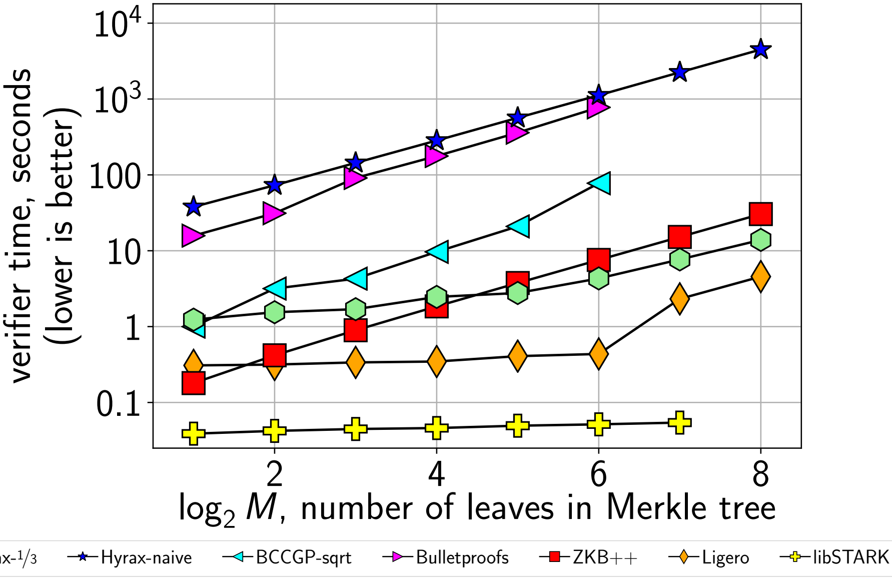
# $\mathcal{P}$ time



# $\mathcal{V}$ time



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`https://hyrax.crypto.fyi`

`https://github.com/hyraxZK`